Interplanetary Travel Lab Guide Experiment A09

Part A: Planetary Orbits and Periods

We know from a previous lab that the eccentricity (e) of a planets orbit is a good way to determine how elliptical it is, i.e., how far away it is form being a perfect circle. An eccentricity of zero corresponds to a perfect circle, and as we increase e closer and closer to 1, we get an orbit that becomes more and more elliptical.

If we know the semi-major axis (a) of an orbit, knowledge of the eccentricity is enough to find both the *perihelion* (point along the orbit that is CLOSEST to the sun) and *aphelion* (point along the orbit that is FARTHEST from the sun) distances. The equations for these distances are quite simple:

$$Perihelion\ Distance\ =\ a(1-e)$$
$$Aphelion\ Distance\ =\ a(1+e)$$

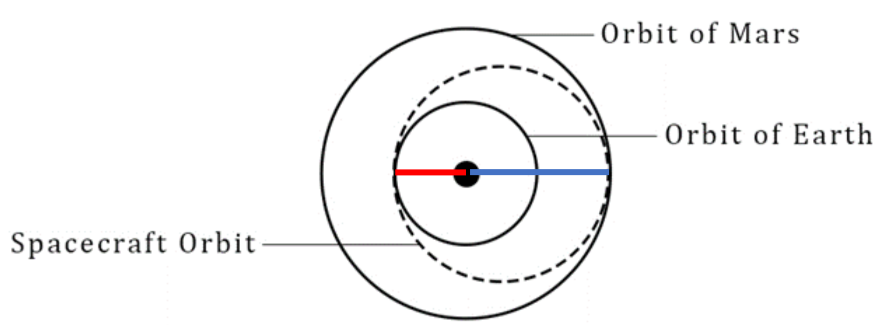
Since e is unitless, the results you get from these equations will be in whatever units the semi-major axis (a) is in.

(20 points) In the table at the end of this section, all you have to do is calculate the perihelion and aphelion distances for 5 planets in our solar system. Since you are given both a and e, you can use the equations above to easily find these distances. Just make sure that since a is in units of AU, so will your answers!

Part B: Transfer Orbits

Space travel is a little bit more complicated than you may expect. We shouldn't just shoot a rocket and travel in a straight path to our destination out in the solar system because that would cost A LOT of energy (money) – thus we need to travel on a more simple path; an ellipse! In this case we use the strong pull of the sun to help us along the path, just like how it helps planets stay in their elliptical paths. We also have the fact that the planets are always in motion; if we are launching a rocket from Earth and our destination is Mars, we must take into account the separate motion of the two planets around the sun.

The easiest way we could get a racket to Mars (for example) is through something called a (Hohmann) *transfer orbit*. In this case, we start with a rocket that has just launched from Earth – meaning it will initially have the same exact path around the sun as Earth has. However, at a certain time we turn on our rocket thrusters and gain a considerable amount of velocity. This increase in velocity (and as a result an increase in total energy E) puts us on a new orbit, one that is more elliptical than the circular orbit of Earth. This orbit is special though. The perihelion of this NEW orbit corresponds with that of Earths orbit, and the aphelion of this NEW orbit corresponds with the orbit of the planet at which we seek to travel to. How much velocity we gain (by firing our rocket boosters) will determine this new perihelion distance, so if we want this distance to correspond exactly to Mars' orbit then we are constrained to gain only a certain amount of velocity from our rocket.



1.) **(6 points)** This question is specifically asking you about the transfer orbit itself – the dotted line in the image above. Notice that this orbit is *not* circular around the sun; gaining velocity has made this new orbit much more elliptical (our eccentricity has increased). However, we fired our rockets (to gain velocity) in a very particular way in order to ensure the perihelion of this new orbit was the same as Earths perihelion radius, and the aphelion of this new orbit was the same as Mars' perihelion radius. Thus, for this transfer orbit, we know exactly what these distances are by looking at the table from section 1 (i.e., we know the perihelion of each planet). The question also asks us to find the period of this new transfer orbit. Kepler's third law says:

$$P^2 = a^3 \quad \text{or} \quad P = \sqrt{a^3}$$

However, we do not, yet, have the semi-major axis of this new orbit! In the image above I included red and blue markers representing the perihelion and aphelion distances, respectively. These two added together make up the *major axis (diameter)* of the ellipse. Thus, *half* of this value would, then, represent the semi-major axis (a) of the orbit. Since you have access to these distances in part 1 in AU's, using the equation above will give you a period (P) in Earth years.

2.) **(6 points)** In the previous problem, we found the period of this new transfer orbit – this would be the time it would take for our spacecraft to make one full orbit around the sun. In this problem, you are asked how long it would take to reach Mars. Remember that Earth is located at perihelion and Mars is located at aphelion of this new orbit; traveling between the two would be traveling along *half* of our full orbit! Thus, if we found how long it would take to travel along one FULL orbit, how long would it take to travel half of that?

3.) **(6 points)** This is a good Google exercise – the problem tells you the day in which you want to reach Mars: March 3, 2029. In the previous problem, you found how long it would take to get from Earth to Mars along this transfer orbit. You need to find the date for which launching a rocket on this day would result in your rocket reaching Mars on March 3, 2029.

4.) **(10 points)** The problem this question is describing is due to the fact that the Earth and Mars themselves are moving along their own particular paths around the sun, so we most likely wouldn't be able to reach Mars when it is exactly at its perihelion. Now, we will reach Mars when it is 1.41 AU away from the Sun – this means our new aphelion distance of our transfer orbit is 1.41 AU (obviously larger than the 1.385 AU aphelion we had before!) This means our major axis (diameter) is *larger* and thus our period is *larger*. You can do the same procedure as before in finding this new period (by first finding the new semi-major axis (a)). Then, you need to find the DIFFERENCE in periods – how much longer does this new trip take (to get to Mars) than it did before?

5.) **(6 points)** In question 4 it was determined you'd launch your rocket on August 1, 2028. Using this new time period you found for this journey, find the new date in which you will reach Mars.

Part C: Velocities

As stated in the lab manual, a planet's speed in its orbit is given by the formula:

$$V = 30 \times \sqrt{\left(\frac{2}{r}\right) - \left(\frac{1}{a}\right)} \left(\frac{km}{s}\right)$$

Where V is the velocity (measured in km/s), a is the semi-major axis of the orbit (in AU's), and r is the exact distance of the planet from the sun at the point you want to find its velocity (also measured in AU's).

1.) **(4 points)** Remember that we are assuming that the Earths orbit is perfectly circular. This means that the distance from the Earth to the sun is always constant and equal to its semi-major axis (i.e., for a circular orbit, its semi-major axis equals its semi-minor axis which are both just the radius of the circle). So here, r and a are going to be the same value, so the equation above simplifies into the following equation:

$$V = 30 \times \sqrt{\left(\frac{1}{a}\right)}\left(\frac{km}{s}\right)$$

And again, if you plug in the value for Earth's semi-major axis (a) in AU's, you will get a value for velocity in km/s. This ends up being very simple since we know a=1.

2.) **(4 points)** Now we know from the previous section that we are NOT assuming Mars has a perfectly circular orbit, so we cannot set r=a as we did in question 1. However, from our table in part A, we know Mars' semi-major axis (in AU's) and we are looking for its velocity when r=1.41 AU. All you have to do is plug these numbers into the first equation above to find the velocity in km/s.

3.) **(4 points)** From looking at the equation for the velocity, we see that all we need to know about the orbit is its semi-major axis and how far the planet (or any object) is away from the sun at the specific time we want. This basically means that if we have an established orbit in mind, we can find the velocity of the object at any point. In this question, we are looking for the *relative* velocity of the spacecraft we're launching to Mars and the Earth. We found the velocity of Earth previously, and since we assumed it to be circular it always will have this velocity. Now we need to find the velocity of the spacecraft. Luckily, we know all about the orbit of our spacecraft – to get to Mars it MUST have the transfer orbit we described in the previous section. Thus, you have found this orbit's semi-major axis (a) in AU's while finding the orbital period. Since we are at the same distance from the Sun as Earth (the problem states that we have JUST escaped Earths gravity), this gives us 'r' at this point. Plugging in this r and a into the velocity equation gives us the velocity our spacecraft must have at this point in its orbit in order to reach Mars. Then, to find the relative velocity, we take this value and subtract from it Earth's velocity. Note that this result you get is still in km/s!

4.) **(4 points)** Now we are looking for the relative velocity between our spacecraft and Mars when we reach the planet. We do the same procedure as before, except now, of course, our 'r' changes in our equation for the velocity of our spacecraft. The velocity of Mars at this point was already calculated in question 2, so to find the relative velocity we just subtract this value from the velocity we find of the spacecraft (is this relative velocity negative?).

Part D: Communicating with Earth

We know communication is not instantaneous because light (the primary way in which we will communicate) does not travel infinitely fast. Because of this, there will be a delay between the time in which we would send a message out (on Mars) and when it is received (back on Earth). Luckily, we assume here that the medium in which this light is propagating in is space – a vacuum – so the speed of light is a constant **$2.99\text{x}10^5$ km/s.**

1.) **(5 points)** Here we are given the maximum distance between Earth and Mars; 2.66 AU. Since we are given the speed of light in km/s, the first thing we should do is convert this distance from AU to km. Then, since we have a distance over which the light is traveling and the velocity at which it travels, we can find the *time* it takes to travel this far using the velocity equation solved for t:

$$velocity\left(\frac{km}{s}\right) = \frac{distance\ (km)}{time(s)} \qquad ; \qquad time\ (s) = \frac{distance\ (km)}{velocity\ \left(\frac{km}{s}\right)}$$

As long as you keep the speed of light (velocity) in km/s and the distance in km, you will get a result in seconds. However, the problem asks for this time in *minutes*; so, you need to convert this result that is in seconds to one that is in minutes.

2.) **(5 points)** The previous question found how long it would take for a message to get from Mars to Earth at a certain distance. Now, we want to know how long it would take for us on Mars to hear a reply (lets assume Earth replies immediately once they get the message). Here we also assume the distance between the planets does not change, so the time it would take for a message to get to Earth from Mars is the same time it would take to get to Mars form Earth. Add the time it takes the message to get from Mars to Earth to the time it takes to get from Earth to Mars and you have the total time it takes to have a reply!

3.) **(5 points)** In this case, we have the same math problem as question 1, but now we have a different *distance* in AU's.

Part E: Return Trip

To get back home to Earth, we have to make another transfer orbit. However, the problem now can be seen in the image on page 3; on our Mars arrival date, the positions of Earth and Mars do not line up with any kind of simple transfer orbit we can make (that also doesn't take a very long time – we have to remember we don't have infinite supplies with us!). This means that unfortunately, we have to wait a little bit on Mars until the two planets arrive in their optimal positions for our return trip.

*NOTE: Planetary orbits are pretty easy for astrophysicists to predict – i.e., we would know exactly when this time is way ahead of time with the help of computers. This is because, for the most part, the only thing acting on these planets is the gravitational force of the sun (and some perturbations from other large planets, but for the most part we can take all that into account with computers).

1.) **(5 points)** Let's see what this question is saying. First, *what is a synodic cycle?* We know that one year on Mars is longer than one year on Earth (we found orbital periods in Earth years back in part A). As a result, from the time we land on Mars, the Earth and Mars will almost always be at different locations along their orbit. However, *one synodic cycle* represents the time it takes until the Earth and Mars are in these *exact same positions relative to one another* again. In this time period, the optimal positioning of the two planets *will occur*, but right now we cannot know exactly when that would be. Thus, we can say that this *one synodic cycle* mark will be the *maximum* time in which it'll take these planets to get into these positions. This time period is what you are looking for and we can find it by the equation given:

$$\frac{1}{P_{syn}} = \frac{1}{P_{Earth}} - \frac{1}{P_{Mars}}$$

Note that this result (when P for the Earth is in years) is one OVER P_{syn}. Thus, to get the actual value P_{syn}, we need to the inverse of this value (HINT: this value should be larger than 1!).

2.) **(5 points)** This problem says that the time it took for the planets to get in this optimal position was ¾ of the time you found in question 1. Thus, the *total time* our journey takes is:

$$total\ time\ =\ time\ to\ Mars\ +\ time\ waiting\ on\ Mars\ +\ time\ to\ Earth$$

You found the travel time to Mars in part B and have just found the time you had to wait on Mars. However, you still need the time it takes to return to Earth. The fortunate part is that all that waiting time was in order for the planets to align themselves so you could take the *same transfer orbit* back to Earth. If this is the *same orbit* as before, then what would *half* of its orbit period be – i.e., the return time to Earth?