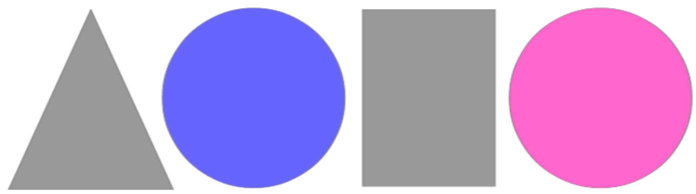
Life in the Universe Lab Guide Experiment A11

Part A: Probability

We are all pretty familiar with probability – we use it to describe how often we should expect a certain outcome after performing some kind of *experiment* multiple times. We can also think of probability NOT in the sense of multiple experiments, but rather the likelihood of a single experiment resulting in a specific outcome (this is probably the way of thinking that most of you are most familiar with). Let's say we have a bag full of 4 colorful shapes: a gray triangle, a blue circle, a gray square, and a pink circle (shown below). The single *experiment* we will do is to pick a single one of these shapes out of the bag without looking. What is the probability that we will pick the blue circle?



Notice that the outcome we desire is the blue circle, and there are 4 different outcomes we could possibly get in this experiment. A good way of thinking about probability in this sense is the following:

$$Probability = \frac{Number\ of\ outcomes\ you\ want}{Number\ of\ outcomes\ possible}$$

So, the outcome we want is to pick the blue circle – thus the number of outcomes we want is just 1. The total number of possible outcomes is 4 – we have 4 objects of which we could possibly pick from the bag. Thus:

$$Probability = \frac{Number\ of\ outcomes\ you\ want}{Number\ of\ outcomes\ possible} = \frac{1}{4}$$

You have a 1 in 4 (1/4, 25%) probability of choosing the blue circle. Now we want to know the probability of choosing a gray shape – any gray shape. Now the number of outcomes that obey this requirement is 2; we can either pick the gray triangle or the gray square. The total number of options we have is still 4, however. Thus:

$$Probability = \frac{Number\ of\ outcomes\ you\ want}{Number\ of\ outcomes\ possible} = \frac{2}{4} = \frac{1}{2}$$

You have a 1 in 2 (1/2, 50%) probability of choosing a gray shape form the bag.

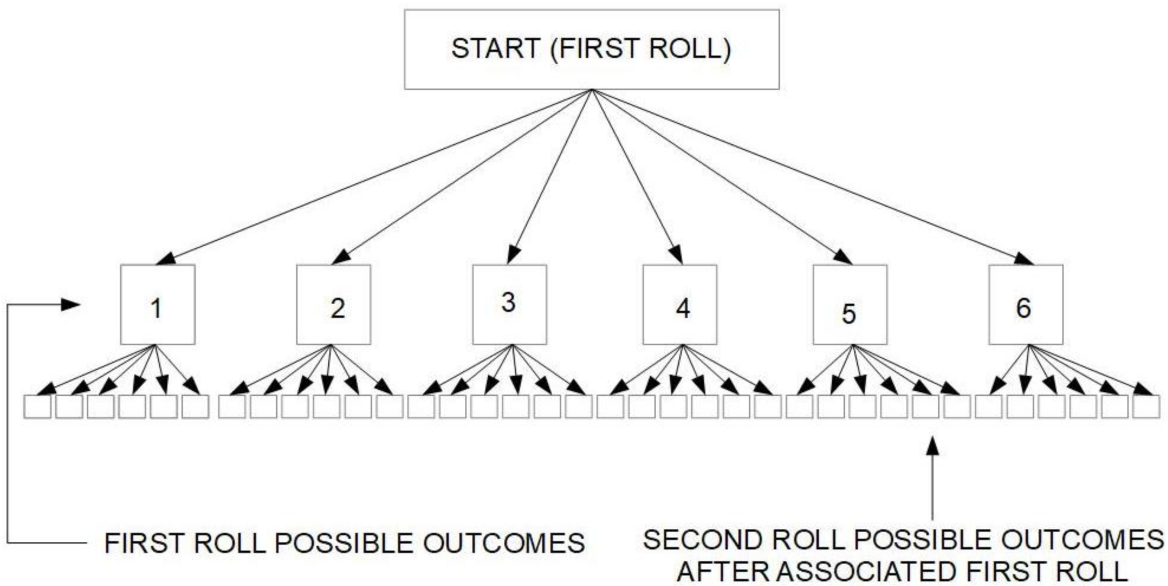
1.) **(2 points)** Now we are considering the probability of rolling a specific number on a die. Using the same logic as above, with our *experiment* being a single roll of a die, we have 6 possible outcomes. However, we are only interested in 1 of those outcomes – the outcome where you roll a 4. So the "Number of outcomes you want" is just 1, and the "Number of outcomes possible: is 6.

2.) **(2 points)** Now here you should actually take a real die (or one on some internet dice-rolling simulator) and record 60 rolls. Mark all of the times you roll a "4" and after 60 rolls count how many "4"'s you rolled. You will get a number that is neither "correct" nor "incorrect" – it is a number that you found and is thus correct in your own experiment. It may or may not be the number we would be *expecting* – how do we know how many times we *expect* to get a specific outcome? Remember we said earlier that *probability* is a way for us to describe how often we would expect to obtain a certain outcome if we performed the same experiment over and over. In this case, the experiment we are performing over and over is the rolling of the die – 60 times to be precise. If we found the probability to roll a "4" in the previous question to be 1/6, then we should *expect* 1/6 of all of our experiment (rolls) to result in a "4"! However, this way of thinking of probability is really only useful when we get to testing a large number of similar experiments. Let's see – how close did you get to the expected number of "4" rolls?

3.) **(2 points)** Here we use the exact same logic as in question 2 when we discussed finding the *expected* number of rolls. In this case, we do 600 rolls (experiments). From question 1, we know that the probability of rolling a "4" is 1/6 (which is the same as saying here that for a large number of experiments we should expect 1/6 of them to be "4"). Thus, 1/6 of the total number of rolls (600) are expected to be "4."

4.) **(3 points)** We have pretty much discussed two definitions of probabilities. You can either elaborate on one of these or write your own.

5.) **(3 points) Compound Probabilities** are a bit more complicated than what we were doing before, but from first principles it works the exact same way. Let's first look at this problem (question 5) by drawing out all of the possible outcomes:



Above, you can see we start with our first roll. The first roll can result in 6 different outcomes. Whatever the outcome we get from our first roll, we can then roll it again and get one of the possible 6 outcomes once more. However, the specific outcome we are looking for is the number "4" followed by the number "2" in a second roll. This is *one specific outcome that we want*. How many total possible outcomes are possible? We can see that this number is represented by the total number of boxes in the bottom row – if you count them then there are 36 possible outcomes here. Thus, using the logic from above:

$$Probability\ of\ rolling\ "4"\ then\ "2"\ = \frac{Number\ of\ outcomes\ you\ want}{Number\ of\ outcomes\ possible} = \frac{1}{36}$$

We see that we have a 1 in 32 chance of rolling the number "4" followed directly by another roll of the number "2".

However, it wouldn't be worth our time to have to draw one of these trees every time we want to solve a compound probability, especially when we get to larger problems. How can we solve this problem much quicker? Let's break up this problem into two – the first roll and then the second roll. Since for each roll individually we only desire one specific outcome of the 6 available, each has a probability of 1/6 of getting the roll we want; the first roll has a 1/6 probability of getting a "4" and the second roll has its own 1/6 probability of getting a "2". So how do we get the *combined* probability? All we have to do is *multiply* the composite probabilities:

$$Probability\ of\ rolling\ "4"\ then\ "2"\ =\ Prob.of\ "4"\ on\ first\ \times\ Prob.of\ "2"\ on\ second\ = \frac{1}{6}\times\frac{1}{6} = \frac{1}{36}$$

See that we get the same probability as we got from counting the results from our tree diagram.

6.) **(3 points)** Now we have a similar problem to the one above, except now we need to pick 3 numbers that can range from 0-9 (10 options per number we pick). So, this would be equivalent to drawing a tree diagram with 10 boxes after the first roll, then each one of those having 10 boxes after our second roll, THEN each one of those having 10 boxes representing our third roll (see why we don't want to draw tree diagrams every time?). We need to pick the exact 3 numbers in order to win. Just like with the dice we can split this up into sections; 3 to be exact. Each section will have its own probability – we see that we have a 1/10 chance to pick the correct first number, and this is also the correct probability for the other two lottery numbers separately as well. Thus, we can get the total probability:

$$\text{Probability of choosing correct lottery numbers } = \frac{1}{10}\times\frac{1}{10}\times\frac{1}{10} = \frac{1}{1000}$$

As most of us know from experience, the probability is a very small number.

7.) **(5 points)** This is a real world example of how you can predict something by estimating certain things related to the problem at hand. Here we will estimate the total number of carpeted rooms in the houses of all of the students in your lab. The first thing we would want to know is probably how many people are in your lab – the number of carpeted rooms in the households of all of your classmates should most definitely depend on the number of people. Then, of course, it'd make sense that this number would depend on the average number of total rooms in a persons house as well as the fraction of those rooms that are carpeted (note that these two numbers must be estimated – you can take a small sample size of people and determine a good number for the total number of rooms, or you could make an educated guess). So, for example, as a first step we could write:

$$\#\ carpeted\ rooms\ =\ (\#\ of\ students) \times (avg.\#\ \text{rooms}) \times (fraction\ of\ carpeted\ rooms) \times \cdots$$

8.) **(5 points)** Remember we said that some of things above you'd have to estimate – if we took a sample of students (i.e., your group members), the logical way to get a more accurate average would just be to get *more people.* Remember, if you were just to go around and ask everyone how many carpeted rooms everyone had, you'd get the *exact* answer.

Part B: The Drake Equation

The Drake equation is an equation similar to the one we made up in question 7 above; it predicts something by multiplying a bunch of relevant numbers together. This is a very underwhelming explanation for something that is actually quite interesting. By watching the YouTube video with the link provided in your lab manual, you will gain a much better understanding of what exactly this equation sets out to do without having to read a novel!

1.) **(16 points)** The Drake equation contains 8 different variables: "N", "R*", all the way to L. Using the video or any online resource, identify each of these variables and give a short description of each.

2.) **(4 points)** Using the calculator on the website linked in the lab manual, you will estimate the values of each one of the variables in the Drake equation in order to come up with N – your estimation of how many intelligent civilizations are present in the Milky Way galaxy. There are no incorrect answers here, just try to come up with reasonable values for these variables form your own knowledge.

3.) **(5 points)** Now you will look for the *actual* values of these variables as agreed upon by the top scientists in these fields. Was your estimation (i.e., your value for N) similar to that found using the *actual* values in the Drake equation?

Part C: The Fermi Paradox

We have a problem: if we believe there to be many intelligent, curious civilizations out there just in our galaxy (the number we found in part B above using the Drake equation), *why have we not found evidence of them?* This is the *Fermi Paradox.* Nobody knows the answer to this question, but I want to know what *you* believe – I want to know why, in your opinion, haven't we found evidence or been in contact with intelligent alien life? If you already have a belief of why, then go ahead and elaborate in your lab manual. If you need to see what others hypothesize, or just are open minded to other ideas, then check out the video linked in the lab manual. Also, use this opportunity to discuss with your friends and/or lab partners. If you all have differing opinions, then you should discuss them and write them both in your lab report.

(50 points) Description/elaboration of your opinion on the Fermi Paradox.